



The Groenewold - Moyal Plane and its Quantum Physics

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Quantum theories constructed on the noncommutative spacetime called the Groenewold-Moyal(GM) plane exhibit many interesting properties such as causality violation, Lorentz and CPT non-invariance and twisted statistics. Such violations lead to many striking features that may be tested experimentally. Thus these theories predict Pauli-forbidden transitions due to twisted statistics, anisotropies and acausal effects in the cosmic microwave background radiation in correlations of observables and Lorentz and CPT violations in scattering amplitudes. Such features of quantum physics on the GM plane are surveyed in this review.

I. INTRODUCTION

Physics at the Planck scale can be radically different. This idea is not new and is attributed to Heisenberg, Pauli and Schrödinger. Later Snyder developed these ideas and published the first paper on this subject [1]. It should be noted that Riemann too had similar visions in the late 19th century when he developed his geometry. Quantum gravity [2] and string theory also indicate that at very small length scales, spacetime could be noncommutative.

We here consider a particular model of such a noncommutative algebra on \mathbb{R}^{d+1} called the Groenewold-Moyal(GM) plane, $\mathcal{A}_\theta(\mathbb{R}^{d+1})$. This algebra does not admit the naive action of the Lorentz group. But it does admit a certain twisted action as we will explain. It results in twisted statistics [3, 4] and generalizes the idea of symmetrized and anti-symmetrized states. These effects can be seen already at the level of quantum mechanics of multi-particle systems itself, though they go on to affect quantum field theories on the GM plane [5], affecting causality and making them nonlocal theories.

In this paper we review these developments and also suggest some future projects. The paper is organized as follows. In section 1 we discuss the notions of causality and statistics in theories on commutative spacetimes. In section 2 we describe the GM plane and discuss the Moyal star product of functions on this non-commutative plane. Section 3 shows how the co-product of spacetime symmetries gets twisted. The consequence of this twist on the statistics of multi-particle states is developed in section 4. In section 5 we formulate twisted quantum theories on the GM plane. Sections 6, 7 and 8 deal with phenomenology on the GM plane illustrating Pauli forbidden transitions, anisotropies in the CMB spectrum and violation of CPT. We then present our conclusions in section 9.

II. CAUSALITY AND STATISTICS

A crucial ingredient in establishing the connection between spin and statistics in local quantum field theories is the requirement of causality [6], [7], [8]. This condition of locality, causality is expressed in such theoretical frameworks by the assumption that the observables localized at spacelike separated regions commute.

But causality plays a role at much more simple levels. In the theory of response functions in physical systems, the Kramers-Kronig relations connect the real and imaginary parts of the response function by making use of the fact that causality implies analyticity and vice versa. This is perhaps the simplest context in non-relativistic physics where causality makes its appearance. We first recall the derivation of this relation.

A physical system should not respond before the time at which it is disturbed. Hence if $R(t)$ is the

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response and disturbance of the system is zero for time $t < 0$,

$$R(t) = 0, \quad t < 0. \quad (1)$$

Consider its Fourier transform

$$\tilde{R}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} R(t) = \int_0^{\infty} dt e^{i\omega t} R(t) \quad (2)$$

For $\text{Im } \omega > 0$, the integral in Eq.(2) converges better because of the extra damping factor $e^{-t\text{Im}\omega}$, the integral in t being from 0 to ∞ . Using this fact, one argues that $\tilde{R}(\omega)$ is holomorphic for $\text{Im}\omega > 0$. This leads to the Kramers-Kronig relations

$$R_1(\omega) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{+\infty} \frac{R_2(\omega')}{\omega' - \omega} d\omega' \quad (3)$$

$$R_2(\omega) = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{+\infty} \frac{R_1(\omega')}{\omega' - \omega} d\omega' \quad (4)$$

where $R_1(\omega)$ and $R_2(\omega)$ denote the real and imaginary parts of the response function $R(\omega)$ and \mathcal{P} denotes the Cauchy principal value.

The real and imaginary parts of the response function have physical interpretations. The imaginary part describes the way the system dissipates and the real part gives information on scattering amplitudes.

The Kramers-Kronig relations are the forerunners of dispersion relations in quantum field theories.

Causal set theory, a discrete approach to quantum gravity, argues that causality is a “partial order” and is based on the central hypothesis that spacetime is a partially ordered or a causal set [9], [10], [11]. In a causal set C , the binary (partial order) condition \succ between its two elements x and y reads: “ $x \succ y$, if x is to future of y .”

There exists a causality condition for the S-matrix called the Bogoliubov-Shirkov causality condition [6]. This is used in axiomatic field theory where the S -matrix is a functional of a function g which is defined as $g : M \rightarrow [0, 1]$, where M is Minkowskian space. This function measures the amount of interaction switched on in the action. For $g(x) = 0$ there is no interaction and for $g(x) = 1$ the entire interaction is switched on. For values of g in between the extreme values, the interaction is only partially switched on. By multiplying the interaction Lagrangian density $\mathcal{L}(x)$ with $g(x)$ we make the new action dependent on interaction with intensity $g(x)$. This makes the scattering matrix a function of g . If x and y are two space-time points and if $x \leq y$ implies that x causally precedes y , then the Bogoliubov-Shirkov causality condition is

$$\frac{\delta}{\delta g(x)} \left(\frac{\delta S(g)}{\delta g(y)} S^\dagger(g) \right) = 0. \quad (5)$$

In local quantum field theories, two observable fields $\rho(x)$, $\eta(y)$ commute if x and y are spacelike separated:

$$[\rho(x), \eta(y)]_- = 0 \quad (6)$$

if $(x^0 - y^0)^2 - (\vec{x} - \vec{y})^2 < 0$, that is, $x \sim y$.

These relations are implemented in quantum field theories by fields. They need not be observable fields. For scalar fields the above relation implies that

$$[\varphi(x), \chi(y)]_- = 0 \quad x \sim y, \quad (7)$$

and for spinor fields it implies that

$$\left[\psi_\alpha^{(1)}(x), \psi_\beta^{(2)}(y) \right]_+ = 0 \quad x \sim y, \quad (8)$$

where \pm denote anticommutator and commutator respectively. As these relations also express statistics we see that causality and statistics are connected.

The above discussion shows that there are different notions of causality in physics. Their relation is not always clear.

It is interesting to study how the connection between causality and statistics is affected in quantum field theories that exhibit features like nonlocality, Lorentz noninvariance etc. A quantum field theory based on the noncommutativity of spacetime shows these interesting features. We can model such spacetime noncommutativity using the algebra of functions called the Groenewold-Moyal (GM) plane. The GM plane describes noncommutative spacetime where commutation relations and hence causality and statistics are deformed.

III. THE GROENEWOLD-MOYAL PLANE

The GM plane is the algebra \mathcal{A}_θ of smooth functions on \mathbb{R}^{d+1} with a twisted (star) product. It can be written as

$$f \star g := m_\theta(f \otimes g)(x) = m_0(\mathcal{F}_\theta f \otimes g)(x) \quad (9)$$

where $m_0(f \otimes g)(x) := f(x) \cdot g(x)$ stands for the usual pointwise multiplication of the commutative algebra \mathcal{A}_0 ,

$$\mathcal{F}_\theta = \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu \otimes \partial_\nu\right), \quad (10)$$

is called the Drinfel'd twist and $\theta^{\mu\nu} = -\theta^{\nu\mu} = \text{constant}$.

The star product implies the commutation relation

$$(\hat{x}_\mu \star \hat{x}_\nu - \hat{x}_\nu \star \hat{x}_\mu) = [\hat{x}_\mu, \hat{x}_\nu]_\star = i\theta_{\mu\nu}, \quad \mu, \nu = 0, 1, \dots, d, \quad (11)$$

with $\hat{x}_\mu =$ coordinate functions, $\hat{x}_\mu(x) = x_\mu$.

We will describe a particular approach to the formulation of quantum field theories on the GM plane and indicate its physical consequences.

It is interesting to see how a noncommutative structure of spacetime emerges at very small length scales from the arguments based on Heisenberg's uncertainty principle and Einstein's theory of classical gravity. Doplicher, Fredenhagen and Roberts [2] give the following arguments.

In order to probe physics at the Planck scale L , the Compton wavelength \hbar/Mc of the probe must fulfill $\frac{\hbar}{Mc} \leq L$ or $M \geq \frac{\hbar}{Lc} \simeq \text{Planck mass}$. Such high mass in the small volume L^3 will strongly affect gravity and can cause black holes and their horizons to form. This suggests a fundamental length limiting spatial localization indicating space-space noncommutativity.

Similar arguments can be made about time-space noncommutativity. Observation of very short time scales requires very high energies. If we also try to probe very short length scales at the same time, then, as in the argument above, they can produce black holes and black hole horizons will then once more limit spatial resolution suggesting $\Delta t \Delta|\vec{x}| \geq L^2$, $L \simeq \text{Planck length}$.

The GM plane *models* such spacetime uncertainties.

IV. THE TWISTED COPRODUCT

If there is a symmetry group G with elements g and it acts on single particle Hilbert spaces \mathcal{H}_i by unitary representations $g \rightarrow U_i(g)$, then conventionally it acts on $\mathcal{H}_1 \otimes \mathcal{H}_2$ by the representation

$$g \rightarrow [U_1 \otimes U_2](g \times g). \quad (12)$$

The homomorphism $\Delta : G \rightarrow G \times G$, $g \rightarrow \Delta(g) := g \times g$ underlying these equations is said to be a coproduct on G .

The action of G on multiparticle states involves more than just group theory. It involves the coproduct Δ .

The \star -multiplication between two functions f and g on the noncommutative algebra can be expressed in terms of the *twist element*, Eq.(10) [12], [13], [14] as

$$f \star g = m_0 \cdot \mathcal{F}_\theta(f \otimes g). \quad (13)$$

Let Λ be an element of the connected component of the Poincaré group \mathcal{P}_+^\dagger . Then for $x \in \mathbb{R}^N$, we have

$$\Lambda : x \rightarrow \Lambda x \in \mathbb{R}^N. \quad (14)$$

It acts on functions on \mathbb{R}^N by pull-back:

$$\Lambda : \alpha \rightarrow \Lambda^* \alpha, \quad (\Lambda^* \alpha)(x) = \alpha[\Lambda^{-1} x]. \quad (15)$$

The work of Aschieri *et al.* [15] and Chaichian *et al.* [14] based on Drinfel'd's original work [13] shows that \mathcal{P}_+^\dagger acts on $\mathcal{A}_\theta(\mathbb{R}^N)$ compatibly with m_θ if its coproduct is “twisted” to Δ_θ where

$$\Delta_\theta(\Lambda) = \mathcal{F}_\theta^{-1}(\Lambda \otimes \Lambda) \mathcal{F}_\theta. \quad (16)$$

V. THE TWISTED STATISTICS

The action of the twisted coproduct is not compatible with standard statistics. Statistics also should be twisted in quantum theory.

A two-particle system for the commutative case ($\theta^{\mu\nu} = 0$) is a function of two sets variables and it lives in $\mathcal{A}_0 \otimes \mathcal{A}_0$. It transforms according to the usual coproduct Δ_0 .

Similarly in the noncommutative case, a state vector lives in $\mathcal{A}_\theta \otimes \mathcal{A}_\theta$ and transforms according to the twisted coproduct Δ_θ .

For $\theta^{\mu\nu} = 0$, we require that the physical wave functions describing identical particles are either symmetric (bosons) or antisymmetric (fermions).

That is, we work with either the symmetrized or antisymmetrized tensor product

$$\phi \otimes_{S,A} \chi \equiv \frac{1}{2} (\phi \otimes \chi \pm \chi \otimes \phi). \quad (17)$$

The equation (17) is Lorentz invariant.

Similarly in a Lorentz-invariant theory, such relations have to hold in all frames of reference. But the twisted coproduct action of the Lorentz group is not compatible with the usual symmetrization/antisymmetrization. We now discuss this point.

Let τ_0 be the statistics (flip) operator associated with exchange for $\theta^{\mu\nu} = 0$:

$$\tau_0(\phi \otimes \chi) = \chi \otimes \phi. \quad (18)$$

For $\theta^{\mu\nu} = 0$, we have the axiom that τ_0 is superselected. In particular, for Lorentz group action, $\Delta_0(\Lambda) = \Lambda \otimes \Lambda$, must and *does* commute with the statistics operator:

$$\tau_0 \Delta_0(\Lambda) = \Delta_0(\Lambda) \tau_0. \quad (19)$$

Hence given an element $\phi \otimes \chi$ of the tensor product, the physical Hilbert spaces can be constructed from the elements

$$\left(\frac{1 \pm \tau_0}{2} \right) (\phi \otimes \chi), \quad (20)$$

each of them being Lorentz invariant.

Now $\tau_0 \mathcal{F}_\theta = \mathcal{F}_\theta^{-1} \tau_0$ so that $\tau_0 \Delta_\theta(\Lambda) \neq \Delta_\theta(\Lambda) \tau_0$. This shows that the usual statistics is not compatible with the twisted coproduct.

But the new statistics operator [16]

$$\tau_\theta \equiv \mathcal{F}_\theta^{-1} \tau_0 \mathcal{F}_\theta, \quad \tau_\theta^2 = \mathbf{1} \otimes \mathbf{1} \quad (21)$$

does commute with the twisted coproduct Δ_θ :

$$\Delta_\theta(\Lambda) = \mathcal{F}_\theta^{-1} \Lambda \otimes \Lambda \mathcal{F}_\theta. \quad (22)$$

The states constructed according to

$$\phi \otimes_{S_\theta, A_\theta} \chi \equiv \left(\frac{1 \pm \tau_\theta}{2} \right) (\phi \otimes \chi), \quad (23)$$

form the physical two-particle Hilbert spaces of (generalized) bosons and fermions respectively and obey twisted statistics.

VI. TWISTED QUANTUM FIELDS

A consequence of twisting the co-product of the Poincaré group is twisted statistics which in turn reflects on the commutation(anti-commutation) relations of boson(fermion) fields in quantum field theory.

A quantum field is an operator-valued distribution acting on a Hilbert space. We can create a particle localized at a point x_1 by acting with a quantum field at x_1 on the vacuum. In a similar way a two-particle state centered at x_1 and x_2 is created by acting with the product of two quantum fields at x_1 and x_2 on the vacuum.

Consider a free scalar field, ϕ_0 of mass m . We can construct the following two-particle state:

$$\begin{aligned} \langle o | \phi_0(x_1) \phi_0(x_2) c_{\mathbf{q}}^\dagger c_{\mathbf{p}}^\dagger | 0 \rangle &= (1 \pm \tau_0)(e_{\mathbf{p}} \otimes e_{\mathbf{q}})(x_1, x_2) \\ &\equiv \langle x_1, x_2 | p, q \rangle_{S_0, A_0}. \end{aligned} \quad (24)$$

Here $\phi_0(x)$ has the mode expansion

$$\phi_0(x) = \int d\mu(p)(c_{\mathbf{p}} e_p(x) + d_{\mathbf{p}}^\dagger e_{-p}(x)) \quad (25)$$

where $e_p(x) = e^{-ip \cdot x}$, $p \cdot x = p_0 x_0 - \mathbf{p} \cdot \mathbf{x}$, $d\mu(p) = \frac{1}{(2\pi)^3} \frac{d^3 p}{2p_0}$, $p_0 = \sqrt{\mathbf{p}^2 + m^2}$. The creation and annihilation operators satisfy the standard commutation(anti-commutation) relations,

$$c_{\mathbf{p}} c_{\mathbf{q}}^\dagger \pm c_{\mathbf{q}}^\dagger c_{\mathbf{p}} = 2p_0 \delta^3(\mathbf{p} - \mathbf{q}), \quad d_{\mathbf{p}} d_{\mathbf{q}}^\dagger \pm d_{\mathbf{q}}^\dagger d_{\mathbf{p}} = 2p_0 \delta^3(\mathbf{p} - \mathbf{q}). \quad (26)$$

The two-particle states in non-commutative quantum field theory should obey twisted statistics. Using Eq.(25) as the guiding principle we can construct the twisted scalar quantum field $\phi_\theta(x)$ as

$$\phi_\theta(x) = \int d\mu(p)(a_{\mathbf{p}} e_p(x) + b_{\mathbf{p}}^\dagger e_{-p}(x)) \quad (27)$$

The creation and annihilation operators defined in the above mode expansion are twisted as we shall see below.

Using the above twisted field we can construct two-particle states as in Eq.(24):

$$\begin{aligned} \langle o | \phi_\theta(x_1) \phi_\theta(x_2) a_{\mathbf{q}}^\dagger a_{\mathbf{p}}^\dagger | 0 \rangle &= (1 \pm \tau_\theta)(e_{\mathbf{p}} \otimes e_{\mathbf{q}})(x_1, x_2) \\ &\equiv \langle x_1, x_2 | p, q \rangle_{S_\theta, A_\theta}. \end{aligned} \quad (28)$$

From these relations we get the relation (see [5] for more details)

$$a_{\mathbf{p}}^\dagger a_{\mathbf{q}}^\dagger = \pm e^{ip_\mu \theta^{\mu\nu} q_\nu} a_{\mathbf{q}}^\dagger a_{\mathbf{p}}^\dagger, \quad a_{\mathbf{p}} a_{\mathbf{q}} = \pm e^{ip_\mu \theta^{\mu\nu} q_\nu} a_{\mathbf{q}} a_{\mathbf{p}} \quad (29)$$

It is possible to write the twisted creation and annihilation operators $a_{\mathbf{p}}^\dagger, a_{\mathbf{p}}$ in terms of the untwisted operators in Eq.(25). The transformation connecting the twisted and untwisted creation and annihilation operators is called the “dressing transformation” [17, 18] and is given by

$$a_{\mathbf{p}} = c_{\mathbf{p}} e^{-\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu}, \quad b_{\mathbf{p}} = d_{\mathbf{p}} e^{-\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu}. \quad (30)$$

Here P_μ is the four-momentum operator given by $\int \frac{d^3 p}{2p_0} (c_{\mathbf{p}} c_{\mathbf{p}}^\dagger + d_{\mathbf{p}} d_{\mathbf{p}}^\dagger) p_\mu$. Note here that we have written the four-momentum operator in terms of the untwisted creation and annihilation operators. The expression for the four-momentum in terms of the twisted ones are just got by replacing the untwisted by the twisted ones as $p_\mu \theta^{\mu\nu} P_\nu$ commutes with $c_{\mathbf{p}}, c_{\mathbf{p}}^\dagger$.

We can write the twisted quantum field in terms of the untwisted one with the help of the dressing transformation as

$$\phi_\theta(x) = \phi_0(x) e^{\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} P_\nu} \quad (31)$$

VII. THE PAULI PRINCIPLE

In [19] the statistical potential V_{STAT} between two identical fermions at inverse temperature β has been computed:

$$\exp\left(-\beta V_{\text{STAT}}(\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2)\right) = \langle \vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2 | e^{-\beta H} | \vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2 \rangle, \quad H = \frac{1}{2m}(\vec{\mathbf{p}}_1^2 + \vec{\mathbf{p}}_2^2).$$

Here $|\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2\rangle$ has twisted antisymmetry: $\tau_\theta |\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2\rangle = -|\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2\rangle$. It is explicitly shown not to have an infinitely repulsive core, establishing the violation of Pauli principle, as has been earlier suggested [20].

This result has phenomenological consequences such as Pauli forbidden transitions (on which there are stringent limits). We indicate them below:

Non-Paulian atoms or nuclei are those whose orbitals can be filled with extra electrons or nuclei violating the Pauli principle. As an example, non-Paulian carbon has as its atomic configuration, $1s^3 2s^2 2p^1$. The presence of just a single electron in the outermost shell of non-Paulian carbon makes it behave chemically like boron, whose atomic configuration is $1s^2 2s^2 2p^1$. Thus by searching for non-Paulian carbon atoms in samples of boron, we can get the concentration of the former in the latter. Bounds on these values were found by the NEMO experiments [21].

In the Borexino [22] and SuperKamiokande [23] experiments, the forbidden transitions from $O^{16}(C^{12})$ to $\tilde{O}^{16}(\tilde{C}^{12})$ where the tilde nuclei have an extra nucleon in the filled $1S_{1/2}$ level are found to have lifetimes greater than 10^{27} years. There are also experiments on forbidden transitions to filled K-shells of crystals done in Maryland which give branching ratios less than 10^{-25} for such transitions. The consequences of these results for noncommutative models are yet to be studied.

VIII. COSMIC MICROWAVE BACKGROUND (CMB)

The COBE satellite, in 1992, detected anisotropies in the CMB radiation, which led to the conclusion that the early universe was not smooth: there were small perturbations in the photon-baryon fluid.

The perturbations could be due to the quantum fluctuations in the inflaton (the scalar field driving inflation). These fluctuations act as seeds for the primordial perturbations over the smooth universe. Thus according to these ideas, the early universe had inhomogeneities and we observe them today in the distribution of large scale structure and anisotropies in the CMB radiation.

The temperature field in the sky can be expanded in spherical harmonics:

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n}). \quad (32)$$

The a_{lm} can be written in terms of perturbations of the Newtonian potential Φ

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3 k}{(2\pi)^3} \Phi(k) \Delta_l^T(k) Y_{lm}^*(\hat{k}), \quad (33)$$

where $\Delta_l^T(k)$ are called the transfer functions.

In a noncommutative spacetime, the quantum corrections to the inflaton fluctuations are modified. The angular correlation functions $\langle a_{lm} a_{l'm'}^* \rangle_\theta$ acquire non-diagonal elements in the magnetic quantum number indicating rotational symmetry breaking in the universe.

It is also θ dependent indicating a preferred direction and are not invariant under rotations. The correlations of a_{lm} are not Gaussian either [24].

On fitting data [25], one finds an upper bound for the length and energy scales associated with spacetime noncommutativity,

$$\sqrt{\theta} \lesssim 10^{-17} \text{ cm}, \quad E \gtrsim 10^3 \text{ GeV}. \quad (34)$$

IX. CAUSALITY, LORENTZ INVARIANCE AND CPT

i) Causality and Lorentz Invariance: The S -matrix of quantum theories constructed on the GM plane is not Lorentz invariant. The underlying reason is loss of causality. Causality is known to be generally required for the Lorentz invariance of the S -matrix [6, 8].

Thus let \mathcal{H}_I be the interaction Hamiltonian density in the interaction representation of the quantum theory. The interaction representation S -matrix is

$$S = T \exp \left(-i \int d^4x \mathcal{H}_I(x) \right). \quad (35)$$

Bogoliubov and Shirkov [6] and then Weinberg [8] long ago deduced from causality (locality) and relativistic invariance that \mathcal{H}_I must be a local field: $[\mathcal{H}_I(x), \mathcal{H}_I(y)] = 0$, $x \sim y$. But noncommutative theories are nonlocal and violate this condition: this is the essential reason for Lorentz noninvariance.

The effect of Lorentz noninvariance on scattering amplitudes is striking. They depend on the total incident momentum \vec{P}_{inc} through the term $\theta^{0i} P_i^{\text{inc}}$.

The effects of $\theta^{\mu\nu}$ disappear in the center-of-mass system, or more generally if $\theta^{0i} P_i^{\text{inc}} = 0$. But otherwise there is dependence on θ_{0i} .

ii) CPT: The noncommutative S -matrix transforms under CPT in the following way [5]:

$$\begin{aligned} S_{\theta}^{M,G} &= T \exp \left[-i \int d^4x \mathcal{H}_{I0}^{M,G}(x) e^{\frac{1}{2} \overleftarrow{\partial} \wedge P} \right] \\ &\rightarrow T \exp \left[i \int d^4x \mathcal{H}_{I0}^{M,G}(x) e^{-\frac{1}{2} \overleftarrow{\partial} \wedge P} \right] = (S_{-\theta}^{M,G})^{-1}. \end{aligned} \quad (36)$$

Here $\mathcal{H}_{I0}^{M,G}$ is the matter-gauge interaction Hamiltonian density for $\theta^{\mu\nu} = 0$. After performing the spatial integration, we can reduce $e^{\frac{1}{2} \overleftarrow{\partial} \wedge P}$ in the S -matrix to $e^{\frac{1}{2} \overleftarrow{\partial}_0 \theta^{0i} P_i}$. Thus the effect of P and CPT is to reverse the sign of θ^{0i} : P or CPT : $\theta^{0i} \rightarrow -\theta^{0i}$. The θ^{0i} contributes to P, and more strikingly, to CPT violation.

The particle-antiparticle life times can differ to order θ^{0i} because of CPT violation:

$$\tau_{\text{particle}} - \tau_{\text{antiparticle}} \cong \theta^{0i} P_i^{\text{inc}}. \quad (37)$$

It can give rise to interesting effects such as mass difference in $K^0 - \bar{K}^0$ system and the difference in $(g-2)$ of μ^{\pm} . (See [27] for bounds on θ estimated from these effects.)

Remark: The decay $Z^0 \rightarrow 2\gamma$ is forbidden even with noncommutativity in the approach of Aschieri *et al.* More generally, a massive particle of spin j does not decay into two massless particles of same helicity if j is odd. This result is the extension of the Landau-Yang theorem [26]. The search for this event is thus of fundamental significance.

X. CONCLUSIONS

Spacetime noncommutativity deforms statistics and so generically violates causality in noncommutative quantum theories. Such effects lead to many interesting features such as *i)* modification of the Pauli principle causing forbidden atomic transitions, *ii)* correlations of observables in spacelike regions giving rise to anisotropies in the CMB radiation, and *iii)* Lorentz and CPT violations in scattering amplitudes.

In this review, we have discussed certain specific observable predictions of these effects and derived bounds on the noncommutativity parameter $\theta_{\mu\nu}$. However more theoretical and experimental work is needed to obtain strong and reliable bounds on $|\theta_{\mu\nu}|$.

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- [2] S. Doplicher, K. Fredenhagen and J. E. Roberts, Phys. Lett. B **331**, 33-44 (1994).
- [3] A.P. Balachandran, A. Pinzul, A.R. Queiroz, Phys. Lett. B 668:241-245 (2008), arXiv:0804.3588 [hep-th].
- [4] A. P. Balachandran, A. Pinzul and B.A. Qureshi, Phys. Rev. D77: 025021 (2008), arXiv:0708.1779 [hep-th].
- [5] E. Akofor, A. P. Balachandran, S. G. Jo and A. Joseph, JHEP **0708**, 045 (2007) [arXiv:0706.1259 [hep-th]].
- [6] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Interscience Publishers, New York, (1959).
- [7] R. Haag, *Local Quantum Physics*, Springer Verlag, Berlin, (1992).
- [8] S. Weinberg, *The Quantum theory of fields. Vol. 1: Foundations*, Cambridge University Press, UK (1995).
- [9] J. Henson, [arXiv:gr-qc/0601121].
- [10] F. Dowker, Contemp. Phys. **47**, 1 (2006).
- [11] R. D. Sorkin, [arXiv:gr-qc/0309009].
- [12] A. P. Balachandran, A. Pinzul, B. A. Qureshi and S. Vaidya, Phys. Rev. D **77**, 025020 (2008) [arXiv:0708.1379 [hep-th]].
- [13] V. G. Drinfel'd, Leningrad Math. J. 1 (1990), 321-332, Leningrad Math. J. **1**, 1419-1457 (1990).
- [14] M. Chaichian *et al.*, Phys. Lett. **B 604**, 98 (2004), [arXiv:hep-th/0408069]. M. Chaichian, P. Presnajder and A. Tureanu, Phys. Rev. Lett. 94, 151602 (2005), [arXiv:hep-th/0409096].
- [15] P. Aschieri *et al.*, Class. Quant. Grav. **22**, 3511 (2005), [arXiv:hep-th/0504183].
- [16] A. P. Balachandran, T. R. Govindarajan, G. Mangano, A. Pinzul, B. A. Qureshi and S. Vaidya, Phys. Rev. D **75**, 045009 (2007) [arXiv:hep-th/0608179].
- [17] H. Grosse, Phys. Lett. B 86, 267 (1979).
- [18] A. B. Zamolodchikov and Al. B. Zamolodchikov, Ann. Phys. 120, 253 (1979); L. D. Faddeev, Sov. Sci. Rev. 0 1 (1980) 107.
- [19] B. Chakraborty, S. Gangopadhyay, A. G. Hazra and F. G. Scholtz, J. Phys. A **39**, 9557 (2006) [arXiv:hep-th/0601121].
- [20] A. P. Balachandran, G. Mangano, A. Pinzul, and S. Vaidya, Int. J. Mod. Phys. A21 : 3111-3126, (2006) [arXiv:hep-th/0508002].
- [21] A. S. Barabash *et al.*, JETP Lett. 68 (1998) 112.
- [22] H. O. Back *et al.* [Borexino Collaboration], Eur. Phys. J. C **37**, 421 (2004)
- [23] Y. Suzuki *et al.* [Kamiokande Collaboration], Phys. Lett. B **311**, 357 (1993).
- [24] E. Akofor, A. P. Balachandran, S. G. Jo, A. Joseph and B. A. Qureshi, JHEP **0805**, 092 (2008) [arXiv:0710.5897 [astro-ph]].
- [25] E. Akofor, A. P. Balachandran, A. Joseph, L. Pekowsky and B. A. Qureshi, Phys. Rev. D **79**, 063004 (2009) [arXiv:0806.2458 [astro-ph]].
- [26] L. D. Landau, Dokl. Akad. Nauk., USSR 60, 207 (1948); C. N. Yang, Phys. Rev. 77, 242 (1950).
- [27] A. Joseph, Phys. Rev. D 79, 096004 (2009) [arXiv:0811.3972 [hep-ph]].